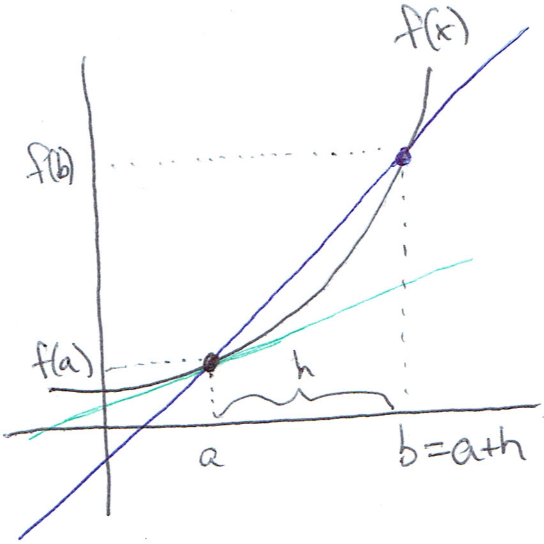
Secant Line vs Tangent line



secant lines go thragh two points on the graph

tangent lines only touch the graph at one point

Slope of the secant line is just the way you always did slope $m = \frac{\Delta y}{\Delta x} \left(= \frac{\text{rise}}{\text{run}}\right)$

$$= \frac{f(b) - f(a)}{b - a} = \frac{f(b) - f(a)}{a + h - a} = \frac{\overset{=f(a+h)}{f(b)} - f(a)}{h}$$

Slope of the tangent line is the derivative at that point: $f'(a)$

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

note: f' "f-prime" is not the same as f^{-1} (f-inverse)

Think of sliding b closer and closer to a (that's what the limit means)

Sometimes we think of the tangent line as the limit of the secant lines

we always want $h \to 0$ because h is the distance between a and b and we want them to get closer and closer together on #19 on the exam:

$f(x) = \sqrt{x}$

a) find the slope of the secant line between $x=16$ and $x=16+h$

$$\frac{\Delta y}{\Delta x} = \frac{f(16+h)-f(16)}{16+h-16} = \frac{\sqrt{16+h}-\sqrt{16}}{h}$$

$$= \frac{\sqrt{16+h}-4}{h}$$

ok to stop here but I'm going to do more rewriting

multiply by conjugate

$$= \frac{\sqrt{16+h}-4}{h} \cdot \frac{(\sqrt{16+h}+4)}{(\sqrt{16+h}+4)}$$

$$= \frac{16+h-16}{h(\sqrt{16+h}+4)}$$

$$= \frac{h}{h(\sqrt{16+h}+4)} = \boxed{\frac{1}{\sqrt{16+h}+4}}$$

b) Take the limit of the slope of the secant line to get the slope of the tangent line

$$\lim_{h\to 0} \frac{1}{\sqrt{16+h}+4} = \frac{1}{\sqrt{16}+4} = \frac{1}{4+4} = \boxed{\frac{1}{8}}$$

* remember we have $h \to 0$ because h is the distance between the x-values of the points we used to draw our secant line.

Lots of people had $h \to 16$. This is wrong because it would mean our points $(16, \sqrt{16}) = (16, 4)$ and $(16+h, \sqrt{16+h})$ wouldn't be getting closer together

#19 cont

c) Find the equation of the tangent line at $x=16$.

- You need two things to find the equation of a line: the slope of the line and a point on the line (could be the intercept)

we found the slope in part b: $m=\frac{1}{8}$

we need a point on the line:

The only one we know for sure is $(16, 4)$ because the tangent line needs to touch $f(x)=\sqrt{x}$ at $x=16$

(we can't use any old point on $f(x)=\sqrt{x}$ because our tangent line doesn't go through all of them)

Let's use point slope form:

$$y-y_1=m(x-x_1)$$

Some people didn't have those parantheses. You need them

$$y-4=\frac{1}{8}(x-16)$$ ✓

rewriting

$$y-4=\frac{1}{8}x-\frac{16}{8}$$

$$y-4=\frac{1}{8}x-2$$

$$y=\frac{1}{8}x+2$$ this is good too